American University of Sharjah
MTH 102
Final Review

Name:—————— **ID:**————————

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1. (5 points) Find the indicated derivative

   (a) $y(x) = \dfrac{1}{2x^2} + 5\dfrac{x^2}{2}$

   (b) $f(x) = \left(x^2 - 1\right)^3 \left(x^3 - 5x + 2\right)$

   (c) $f(x) = 2\sqrt{x} + \dfrac{4}{\sqrt[3]{x^2}}$

   (d) $f(x) = \left(\frac{2x-4}{3x^2-1}\right)^5$

   (e) $f\,(x) = x^2 e^{-4x}$

   (f) $y = \ln\left(3x^2 - 4x + \dfrac{1}{x}\right)$

2. Find each limit, if it exists

   (a) $\lim\limits_{x\to 0} \dfrac{2x}{3x^2 - 2x}$

   (b) $\lim\limits_{x\to 1} \dfrac{3x^2 - x - 2}{x^2 + 4x - 5}$

   (c) $\lim\limits_{x\to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

   (d) $\lim\limits_{x\to 1^-} \dfrac{2 - 2x}{|1 - x|}$

   (e) $\lim\limits_{x\to 2} \dfrac{\sqrt{x-1} - 1}{1 - x}$

   (f) $\lim\limits_{x\to \infty} \dfrac{2x^2 + 10x - 2}{x^2 + 4x^3 - 5}$

   (g) $\lim\limits_{x\to -\infty} \dfrac{x^5 + 2x - 10}{-2x^2 + 10x - 3}$

3. Find the equation of the tangent line

   (a) $y^2 - xy - 6 = 0$ at $x = 1$

(b) $xe^y = 1$ at $x = 0$

4. Use the definition of derivative $\left(f^{\prime}(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right)$ to evaluate the derivative of

   (a) $f(x) = \sqrt{1-4x}$

   (b) $f(x) = -\frac{2}{x^2}$, find $f^{\prime}(1)$

5. Let $p = 25 - 0.01x$ and $C(x) = 2x + 9000, \quad 0 \le x \le 2500$ be the price –demand equation and the cost function respectively, for the manufacture of umbrellas.

   (a) Find the marginal cost, average cost, and marginal average cost functions.

   (b) Express the revenue in terms of , and find the marginal revenue, average revenue, and marginal average revenue functions.

   (c) Find the profit, marginal profit, average profit, and marginal average profit functions.

   (d) Evaluate the marginal profit at $x = 1000$, $x = 1140$ and $x = 1400$ and interpret the results.

6. The price p (in dollars) and the demand x for a particular clock radio are related by the equation

$$x = 4000 - 40p$$

   (a) Express the price p in terms of the demand x and find the domain of this function.

   (b) Find the revenue $R(x)$ from the sale of x clock radios. What is the domain of $R(x)$?

   (c) Find the marginal revenue at a production level of 1600 clock radios and interpret?

   (d) Find the exact revenue from selling the 1601st clock radio? Compare your answer to that in part c).

   (e) Find the average revenue from selling x clock radios?

   (f) Find the marginal average revenue if 1600 radios are sold and interpret?

7. Given the price-demand equation

$$p + 0.01x = 50$$

   (a) Express the demand x as a function of the price p

   (b) Find the elasticity of demand, $E(x)$

(c) What is the elasticity of demand if the price is increased by \$10? If the price is decreased by 5%, what is the approximate change in demand?

(d) What is the elasticity of demand if the price is increased by \$45? If the price is decreased by 5%, what is the approximate change in demand?

8. A fast food restaurant can produce an order of fries for \$0.40. If the restaurant's daily sales are increasing at the rate of 15 per day, how fast is its daily cost of fries increasing?

9. The price-demand equation for an order of fries at a fast-food restaurant is

$$x + 1000p = 1000$$

Currently, the price of an order of fries is \$0.30. If the price is decreased by 10%, will the revenue increase or decrease?

10. A company manufactures and sells x television sets per month. The monthly cost and demand equations are given

$$\begin{aligned} C(x) &= 72000 + 60x \\ p &= 200 - \frac{x}{30} \qquad\qquad 0 \le x \le 6000 \end{aligned}$$

(a) Find the maximum revenue.

(b) Find the maximum profit, the production level that will realize the maximum profit, and the price the company should charge for each television set.

(c) If the government decides to tax the company \$5 for each television set it produces, how many sets should the company manufacture each month to maximize its profit? (Just explain briefly how can you solve it)

11. Find all horizontal and vertical asymptotes

(a) $f(x) = \dfrac{x-1}{x^3+3}$

(b) $f(x) = \dfrac{x^3}{x^2+4x+4}$

(c) $f(x) = \dfrac{x^2-4}{x^2+4}$

12. Graph the following functions by identifying all important components (asymptotes, critical numbers, local extrema, etc.)

(a) $f(x) = x^2 - 2x + 3$

(b) $f(x) = x + \frac{4}{x}$

(c) $f(x) = (x-1)(x-5)^5 + 1$

(d) $f(x) = \dfrac{x-1}{2x+3}$

(e) $f(x) = \dfrac{2x}{x^2-1}$

13. Use the following information to sketch the graph of $f(x)$.

- Domain: $\mathbb{R}$
- $f(-2) = 1, f(0) = 0, f(2) = 1.$
- $f'(0) = 0$
- $f'(x) > 0$ on $(0, \infty)$ and $f'(x) < 0$ on $(-\infty, 0)$
- $f''(-2) = 0$ and $f''(2) = 0$
- $f''(x) > 0$ on $(-2, 2)$ and $f''(x) < 0$ on $(-\infty, -2) \cup (2, \infty)$.
- H. A. $y = 2$

14. Let $p(x) = 400 - 0.4x$ and $c(x) = 160x + 2000$ be the price-demand equation and the cost function respectively for the manufacturing and selling of x digital cameras per week.

(a) Find the price of the camera that will give the maximum revenue.

(b) What is the maximum weekly revenue?

(c) What is the maximum weekly profit?

(d) Find the price that will produce the maximum profit.

(e) If the government decides to tax the company \$4 for each camera it produces, how many cameras should the company manufacture each week to maximize the profit?

15. Suppose that a certain country produces $3,000,000$ barrels of oil every day and sell them at a price of \$55 per barrel. A market survey showed that for every \$1 increase in price 20000 fewer barrels are sold. Find the price of each barrel that will produce the maximum revenue.

16. A car rental agency rents 200 cars per day at a rate of \$30 per day. For each \$1 reduction, 10 more cars are rented. At what rate should the cars be rented to produce the maximum revenue? What is the maximum revenue?

17. Evaluate each of the following integrals

(a) $\displaystyle\int_0^1 x^3\sqrt{x^4+8}\,dx$

(b) $\int x^2 e^{x^3-4} dx$

(c) $\int \frac{e^x - 3x^2}{2} dx$

(d) $\int \left(5\sqrt[3]{x^2} - 4x^{-1} + 2e^2\right) dx$

(e) $\int \left(3t^4 - \frac{2}{t^3} + e^{4t-1}\right) dt$

(f) $\int_0^2 \frac{4x+1}{(4x^2+2x+3)^3} dx$

(g) $\int_1^3 \frac{(\ln x)^4}{x} - 4x dx$

(h) $\int \frac{1}{x(\ln(x))^2} dx$

(i) $\int \frac{x}{\sqrt{x-1}} dx$

18. The marginal price for a weekly demand of x bottles of baby shampoo in a drugstore is given by

$$p'(t) = \frac{-600}{(3x+50)^2}$$

Find the price demand equation if the weekly demand is 150 when the price of a bottle of shampoo is \$4.

19. Find the absolute maximum and minimum for

(a) $f(x) = x^{5/3} + 10x^{2/3}$ on $[-1, 1]$.

(b) $f(t) = t\sqrt{4 - t^2}$ on $[-1, 2]$.

20. Find the absolute maximum for $f(x) = 10xe^{-2x}$ on $(0, \infty)$.

21. Find the consumers' surplus at a price level of $\overline{p} = \$150$ for the price demand equation

$$p = D(x) = 400 - 0.05x.$$

Interpret your results.

22. Find the producers' surplus and interpret your results at a price level of $\overline{p} = 67$ for the price-supply equation

$$p = S(x) = 10 + 0.1x + .0003x^2.$$

23. Find the consumers' surplus and the producers' surplus at the equilibrium price level for the price-demand equation $p = D(x) = 80e^{-0.001x}$ and the price-supply equation $p = S(x) = 30e^{0.001x}$.

24. Find the indicated partial derivative

    (a) $f_y(x, y)$ if $f(x, y) = 3x^2 + 2xy - 7y^2 + 12$
    (b) $f_{xy}(x, y)$ if $f(x, y) = -4x^3y^5 + 9x^3y^{-2} + y^3$
    (c) $f_{xx}(x, y)$ and $f_{yy}(x, y)$ for $f(x, y) = e^{x^2y^3}$
    (d) $f_{xy}(2, 3)$ for $f(x, y) = \dfrac{\ln x}{y}$

25. A firm produces two types of computers each month, x of type A and y of type B. The weekly revenue and cost functions (in dollars) are

$$\begin{aligned} R(x, y) &= 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2 \\ C(x, y) &= 8x + 6y + 20000 \end{aligned}$$

    (a) If $P(x, y)$ is the profit function, find $P_x(x, y)$ and $P_y(x, y)$.
    (b) Evaluate $P_x(1400, 1700)$ and $P_y(1400, 1700)$ and interpret your results.

26. The productivity of an automobile manufacturing company is given approximately by

$$f(x, y) = 50x^{0.5}y^{0.5}$$

with the utilizing of x units of labor and y units of capital.

    (a) Find $f_x(x, y)$ and $f_y(x, y)$.
    (b) If the company is now using 250 units of labor and 125 units of capital, find the marginal productivity of labor and the marginal productivity of capital.
    (c) Should the company encourage the increase use of labor or capital?

27. Find the local extrema if any

    (a) $f(x, y) = x^2 - y^2 + 2x + 6y - 4$
    (b) $f(x, y) = 2x^2 - xy + y^2 - x - 5y + 8$
    (c) $f(x, y) = x^2y - xy^2$
    (d) $f(x, y) = e^{xy}$

28. A firm produces two types of calculators per year: x thousand of type A and y thousand of type B. If the revenue and cost equations for the year in millions of dollars are

$$\begin{aligned} R(x, y) &= 2x + 3y \\ C(x, y) &= x^2 - 2xy + 2y^2 + 6x - 9y + 5 \end{aligned}$$

determine how many of each type of calculators should be produced per year to maximize the profit. What is the maximum profit?

Test One , MTH 102, Spring 2010 at 2 pm

Ayman Badawi

QUESTION 1. (21 points) Find $f'(x)$ and DO NOT SIMPLIFY:

(i) $f(x) = \sqrt[3]{(-3x^3 + 3x^2)^2(4x^2 + 3x - 1)}$

(ii) $f(x) = \frac{3x^2+6x-1}{-7x+11}$

(iii) $f(x) = (5x^2 + 6)^{13} + \sqrt{8x + 2} + \frac{7}{-2x^3+7x-1}$

QUESTION 2. (23 points) Use the concept of Horizontal asymptotes, Vertical asymptotes, and limits to graph (roughly) the function $f(x)\frac{-6x^2-12}{2x^2-8x+6}$ (you may finish on the back)

QUESTION 3. (24 points) Let $P(x) = x\sqrt{3x+6} + \frac{36x}{3x+6}$ be a profit function.

(i) Find the Average Profit function (i.e. find $\overline{P(x)}$).

(ii) Find the equation of the tangent line to the curve of the average profit function at the point $(10, \overline{P(10)})$.

(iii) Find the (exact) average profit (profit per item) on selling 22 items.

(iv) Use (ii) to approximate the average profit on 22 items.

(v) use the concept of marginal average profit to approximate the average profit on 22 items.

QUESTION 4. (10 points) Let $f(x) = \sqrt{4x+5}$. Find $f'(5)$ using the definition of $f'(x)$. (you may use the back)

QUESTION 5. (22 points)

(i) find $lim_{x \to -3} \ \frac{x^2+x-6}{x^2-9}$

(ii) find $lim_{x \to 2} \ \frac{\sqrt{x+7}+3}{-x+4}$

(iii) Let $f(x) \mid 2x - 4 \mid$

a. Write f(x) as a piece-wise function.

b. Sketch the graph of f(x)

c. Find $lim_{x \to 2^-} \ \frac{|2x-4|}{x^2-x-2}$

Faculty information

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Name Asma Al Rafi, ID 36401

TEST Two, MTH 102, Spring 2010 at 11am

Ayman Badawi

100

QUESTION 1. (20 points) Find the first derivative but do not simplify:

(i) $f(x) = ln[(3x+1)^5(-7x+2)^2]$

$f(x) = 5\ln(3x+1) + 2\ln(-7x+2)$

$f'(x) = \frac{15}{3x+1} + \frac{-14}{-7x+2}$ ✓

(ii) $f(x) = 10e^{7x^2+x+1}$

$f'(x) = 10\,(14x+1)\,e^{7x^2+x+1}$ ✓

(iii) $f(x) = -5[e^{x+3} + ln(3x+6)]^7$

$f'(x) = -35\,(e^{x+3} + \ln(3x+6))^6\,(e^{x+3} + \frac{3}{3x+6})$ ✓

(iv) $f(x) = \frac{e^{3x}+x^2}{x^3-x^2+1} = \frac{u}{v}$

$f'(x) = \frac{(3e^{3x}+2x)(x^3-x^2+1) - (3x^2-2x)(e^{3x}+x^2)}{(x^3-x^2+1)^2}$ ✓

QUESTION 2. (10 points) Given: $y^2e^x + 2x + 3yx + y^2 - 8 = 0$. Find y', then find the equation of the tangent line to the curve at $(0, 2)$

$y' = -\dfrac{y^2e^x + 2 + 3y}{2e^xy + 3x + 2y}$

y' at $(0,2) \Rightarrow y' = m = -\dfrac{(2)^2e^0 + 2 + 3(2)}{2e^0(2) + 3(0) + 2(2)} = -\dfrac{4+2+6}{4+4}$

$\boxed{m = -3/2}$

$y = mx + b$

$2 = -\frac{3}{2}(0) + b$ $\boxed{b = 2}$

* Equation of tangent line:

$y = -\frac{3}{2}x + 2$ ✓

QUESTION 3. (20 points) Let x be number of items in tens and $P(x)$ be the profit function in hundreds of dirham such that $P(x) = 0.5x^2 - 120ln(x-2) + 600$. Given $3 \le x \le 20$.

[3, 20]

a) For what values of x does an absolute minimum of profit occur? What is the absolute minimum profit?

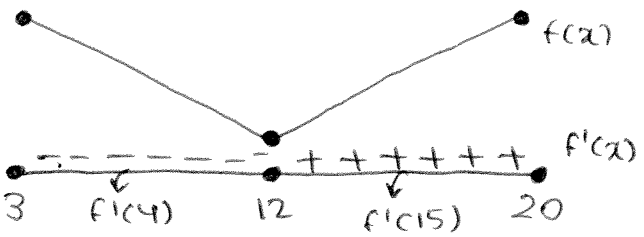
① $P'(x) = x - \dfrac{120}{x-2} = \dfrac{x(x-2) - 120}{x-2} = \dfrac{x^2 - 2x - 120}{x-2} = 0$

② $x^2 - 2x - 120 = 0$

$(x-12)(x+10) = 0$

$x = 12$ $x = -10$ (rejected)

③



④ There is a local and absolute minimum at $\boxed{x = 12}$, at 120 iter[ms]

$f(12) = 0.5(12)^2 - 120\ln(12-2) + 60[0]$

$f(12) = 72 - 120\ln(10) + 60[0]$

$f(12) \approx 395.6898$

$(12, 395.6898) \to$ Abs. Mi[n]

⑤ When we purchase 120 items, profit earned is minimimum and its 395,689.8[0]

b) For what values of x does an absolute maximum profit occur? What is the absolute maximum profit?

①

x	f(x)
3	$= 0.5(3)^2 - 120\ln(3-2) + 600$ $= 604.5$ ⟹ Abs. Max
20	$= 0.5(20)^2 - 120\ln(20-2) + 600$ $= 200 - 120\ln(18) + 600 \approx 453.15$

× 100

② $(3, 604.5) \to$ Abs max.

③ Maximum profit is 60,450 d and it occurs when we purchase 30 items

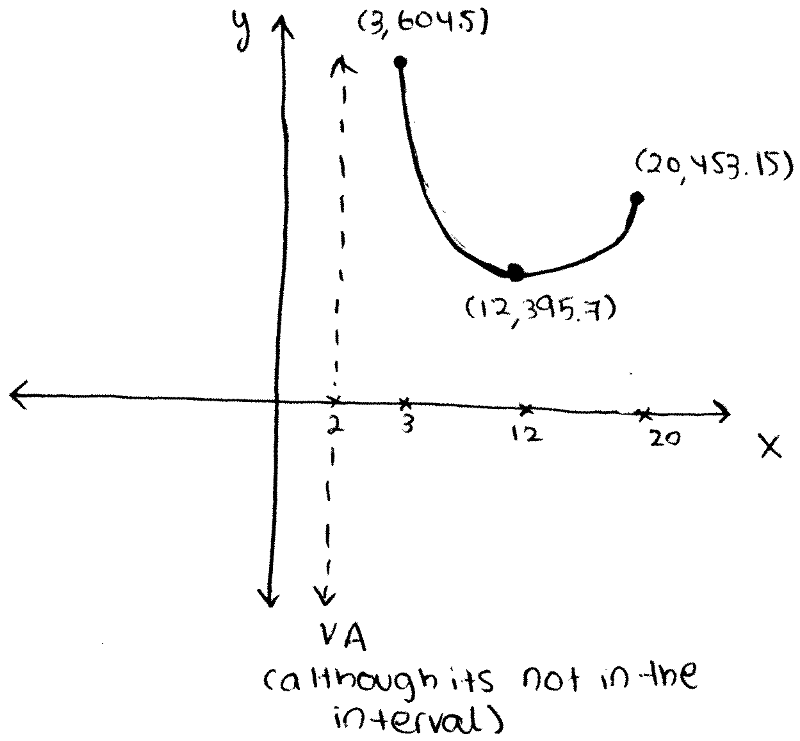
c) Sketch the profit function on the given interval [3, 20]. (Use only the first derivative to do that and you may use the back page to graph)

① $x - 2 = 0$

$\boxed{x = 2}$ ⟹ V.A (although it's not in the interval anyway)

⟹ check behind for graph

② through second derivative, we know the graph should be concave up.


y
(3, 604.5)
(20, 453.15)
(12, 395.7)
2
3
12
20
X
VA
(although its not in the
interval)

QUESTION 4. (20 points) Let $f(x) = 4xe^{-0.5x^2}$ defined on all real numbers. Find all local min values and local max values of $f(x)$ and use only first derivative to sketch f(x) (you may use your calculator to find the y-values to the nearest 2 decimals).

① $f'(x) = 4e^{-0.5x^2} + 4x(-x)e^{-0.5x^2}$

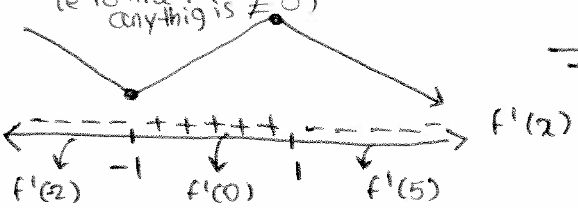
$f'(x) = 4e^{-0.5x^2} - 4x^2 e^{-0.5x^2}$

$f'(x) = 4e^{-0.5x^2}(1 - x^2) = 0$

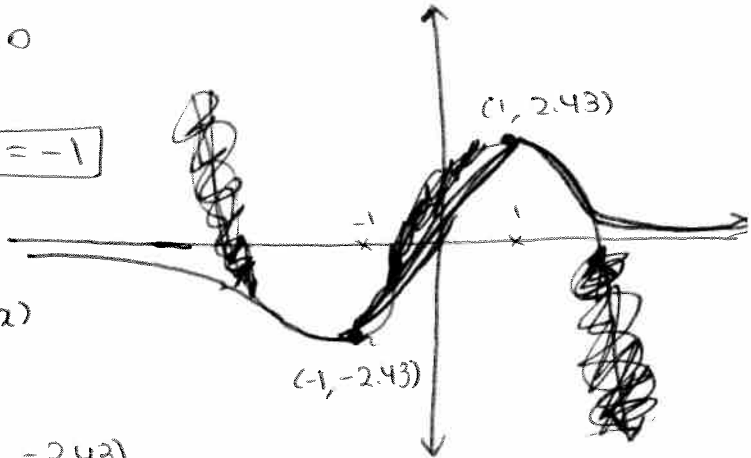
$4e^{-0.5x^2} = 0$ $\qquad 1 - x^2 = 0$

Impossible (e to the power anythig is $\neq 0$) $\qquad$ $x = 1$ $\quad$ $x = -1$

②



⑤ Graph:



③ local minimum at $x = -1$: $(-1, -2.43)$

$f(-1) = 4(-1)e^{-0.5(-1)^2} \approx -2.43$

④ local maximum at $x = 1$ $(1, 2.43)$

$f(1) = 4(1)e^{-0.5(1)^2} \approx 2.43$

QUESTION 5. (10 points) Given $p = -\sqrt{x} + 1000$ is the price per item and x be number of items such that $0 < p \leq 1000$.

a) Find E(p).

① $p = -\sqrt{x} + 1000$

$\sqrt{x} = 1000 - p$

$f(p) = x = (1000 - p)^2$

② $f'(p) = -2(1000 - p)$

③ $E(p) = -\dfrac{p f'(p)}{f(p)}$

$E(p) = -\dfrac{-2p(1000-p)}{(1000-p)^2}$

✓ $E(p) = \dfrac{2p}{1000 - p}$

b) USE (a) to find all values of p where revenue decreases

When Demand is elastic, as price increases, revenue decreases. and demand is elastic when $E(p) > 1$.

$\dfrac{2p}{1000 - p} > 1$

$2p > 1000 - p$

$3p > 1000$

$p > \dfrac{1000}{3}$

✓

conclusion: Revenue is decreasing on: $\left(\dfrac{1000}{3}, 1000\right]$

$\dfrac{1000}{3}$ not included because there is unit elasticity

as $p > \dfrac{1000}{3}$, demand is elastic so: $\left(\dfrac{1000}{3}, 1000\right] \to$ Revenue decreases

QUESTION 6. (20 points) Let x be number of A-Calculators in tens, and y be number of B-Calculators in tens. Given $R(x,y) = 20x + 30y$ and $C(x,y) = 10x^2 - 20xy + 20y^2 + 60x - 90y + 50$ (C(x, y) and R(x, y) are in hundred of dirham). For what values of x and y does a maximum profit occur? What is the maximum profit?

① $P(x,y) = R(x,y) - C(x,y)$

$P(x,y) = 20x + 30y - 10x^2 + 20xy - 20y^2 - 60x + 90y - 50$

$P(x,y) = -40x + 120y - 10x^2 + 20xy - 20y^2 - 50$

② $P_x(x,y) = -40 - 20x + 20y$

$P_y(x,y) = 120 + 20x - 40y$

③ set $P_x(x,y) = 0$ & $P_y(x,y) = 0$ & use elimination to solve:

$40 = -20x + 20y$

(+) $-120 = 20x - 40y$

$-80 = -20y$

$\boxed{y = 4}$

subs. $y=4$ to find x:

$0 = -40 - 20x + 20(4)$

$-40 = -20x$

$\boxed{x = 2}$

$\boxed{(2,4)}$

④ $P_{xx}(x,y) = -20 \rightarrow P_{xx}(2,4) = -20$ —— (A)

$P_{yy}(x,y) = -40 \rightarrow P_{yy}(2,4) = -40$ —— (B)

$P_{xy}(x,y) = P_{yx}(x,y) = 20 \rightarrow P_{xy}(2,4) = P_{yx}(2,4) = 20$ ——

⑤ $AB - C^2 \overset{?}{>} 1$

$(-20)(-40) - (20)^2 \overset{?}{>} 1$

$800 - 400 \overset{?}{>} 1$

$400 > 1$ ✓

* $A < 1$, then we have a local max at $(2,4)$.

⑥ $P(2,4) = -40(2) + 120(4) - 10(2)^2 + 20(2)(4) - 20(4)^2 - 50$

$= 150$

⑦ conclusion: when we purchase 20 of calculator A and 40 of calculator B, we earn maximum profit of 15,000 dhs.

Faculty information

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TEST Two, MTH 102, Spring 2010

Ayman Badawi

QUESTION 1. (20 points) Find the first derivative but do not simplify:

(i) $f(x) = ln[\frac{(3x+1)^5}{(-7x+2)^2}]$

(ii) $f(x) = 2e^{7x^2+x+1}$

(iii) $f(x) = 5[e^{x+3} + ln(3x+6)]^7$

(iv) $f(x) = \frac{2e^{3x}+x^2}{x^3+x^2+1}$

QUESTION 2. (10 points) Given: $y^2e^x + 2x + 3yx + 2y^2 - 12 = 0$. Find y', then find the equation of the tangent line to the curve at $(0, 2)$

QUESTION 3. (20 points) Let x be number of items in tens and $P(x)$ be the profit function in hundreds of dirham such that $P(x) = 0.5x^2 - 140ln(x - 2) + 600$. Given $3 \leq x \leq 20$.

a) For what values of x does an absolute minimum of profit occur? What is the absolute minimum profit?

b) For what values of x does an absolute maximum profit occur? What is the absolute maximum profit?

c) Sketch the profit function on the given interval [3, 20]. (Use only the first derivative to do that and you may use the back page to graph)

QUESTION 4. (20 points) Let $f(x) = 2xe^{-0.5x^2}$ defined on all real numbers. Find all local min values and local max values of $f(x)$ and use only first derivative to sketch f(x) (you may use your calculator to find the y-values to the nearest 2 decimals).

QUESTION 5. (10 points) Given $p = -\sqrt{x} + 2100$ is the price per item and x be number of items such that $0 < p \leq 2100$.

a) Find E(p).

b) USE (a) to find all values of p where revenue decreases

QUESTION 6. (20 points) Let x be number of A-Calculators in tens, and y be number of B-Calculators in tens. Given $R(x, y) = 200x + 300y$ and $C(x, y) = 100x^2 - 200xy + 200y^2 + 600x - 900y + 100$ (C(x, y) and R(x, y) are in hundred of dirham). For what values of x and y does a maximum profit occur? What is the maximum profit?

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Final Exam, MTH 102, Spring 2010

Abu-alrub, Abu-khaled, Abu-Muhanna, Badawi

QUESTION 1. (8 Points) Find the indicated partial derivatives BUT DO NOT SIMPLIFY YOUR ANSWER.

(i) f_{xy} for $f(x, y) = \ln(2x + 3yx)$

(ii) f_{xx} for $f(x, y) = (xe^y + y^2x)^4$

QUESTION 2. (6 Points) Find the first derivative BUT DO NOT SIMPLIFY YOUR ANSWER.

(i) $f(x) = e^{\pi} + \ln(3x + 10) - x^3 - e^{2x}$

(ii) $f(x) = \sqrt[3]{3xe^{2x}}$

QUESTION 3. (5 Points) Use implicit differentiation to find the slope of the tangent line to the graph of $2y^2 + xe^y - 3xy = 1$ at the point $(1, 0)$

QUESTION 4. (8 Points) Find the absolute minimum and the absolute maximum of $f(x) = 10xe^{-0.5x^2+0.5} + 4$ on the interval [-2, 3]

QUESTION 5. (13 Points) The total cost function of producing x units of product A and y units of product B is $C(x, y) = 2x^2 + 3y^2 - 4xy - 100x + 60y + 1000$. Find the values of x and y that produce the minimum cost. Verify your answer. What is the minimum cost?

QUESTION 6. (9 Points) Integrate the following

(i) $\int_0^1 (5x^4 + 3\sqrt{x} - e^{-x})\, dx$

(ii) $\int \frac{x}{10x^2+12}\, dx$

(iii) $\int (2x-3)^6\, dx$

QUESTION 7. (9 Points) The marginal average cost for producing x items of a certain product is given by $[\overline{C(x)}]' = \frac{-40}{x^2}$ and the average cost of producing 10 items is 2 dirhams per item. Find the cost, $C(x)$, of producing 10 items.

QUESTION 8. (12 Points) The demand and supply equations are given by

Demand $p = D(x) = 30 - 0.005x^2$ and Supply $p = S(x) = 5 + 0.005x^2$ where x is the number of items and p is the price per item in dirhams.

(i) Find the equilibrium point.

(ii) Write the consumer surplus as an integral and compute it at the equilibrium point.

QUESTION 9. (6 Points) The demand function for a certain item is $x = \dfrac{150-0.5p^2}{60}$ and $0 < p \leq 18$. For what value(s) of p is Revenue decreasing?

QUESTION 10. (12 Points) A company manufactures and sells televisions per month. The monthly cost and price demand equations are (**Cost**) $C(x) = 180x + 20,000$ and (**Demand: selling price per television**) $p = 220 - 0.001x$, $0 \leq x \leq 100,000$

(i) How many televisions should the company manufacture each month to maximize its monthly profit?

(ii) What is the maximum monthly profit, and what should the company charge for each TELEVISION to realize the maximum monthly profit?

QUESTION 11. (12 Points) Consider the function $f(x) = -x^3 + 12x + 1$.

(i) Find the increasing and decreasing intervals, local extrema (local min and local max) , the concave upward and downward intervals.

(ii) Use your findings in part **a** to sketch the graph of $f(x)$

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